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University of Virginia PO Box 400137, Kerchof Hall, 309 Charlottesville, VA	

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Dynamics and Control Dr. Fariba Fahroo AFOSR 875 N Randolph St, Arlington, VA 22203	
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14. ABSTRACT

The PIs, along with their PhD students and collaborators (one at the Air Force Academy in Colorado) have solved numerous problems of the proposed research project. These are described in the Final Report.

15. SUBJECT TERMS

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Analysis and Control of Fluids, Waves, Material Structures and Their Interaction

Over the duration of said grant, the PIs—along with their graduate PhD students and collaborators—have been successfully engaged on several fronts of the proposed research project. These efforts have produced quite a few papers, too many to be listed here. Accordingly, they are referred to the PIs' publicly available CVs in the appropriate box. Instead, here, we provide two broad sets of results: (A) Theoretical results with special attention paid to PhD theses, which have resulted from these projects under the grant; (B) Numerical results, either illustrating the theoretical results or else complementing the theoretical results, again performed by PhD students as part of these theses. Emphasis on PhD theses that have originated from the general research topic of the grant is explicitly noted in the instructions to the Final Report.

(A) Theoretical Results (to include PhD theses). Some of the main topics investigated under the grants include:

(a) Fluid-structure interaction. Here a structure, modeled by the system of dynamic elasticity is surrounded by a fluid modeled by the full Navier-Stokes equation (or its linearization). The interaction takes place at the interface between the two media. Control, disturbance, damping may be active precisely at the interface. In the absence of external forces, the mathematical coupling is hyperbolic (structure)–parabolic (fluid). Restrictions (traces) of the structure solution on the interface is a very delicate issue and provides the boundary input to the surrounding fluid. Problems investigated successfully include well-posedness at the state space level of weak solutions; regularity at a higher energy level of strong solutions; strong stability of the original model, as well as uniform stability of the model supplemented by damping at the interface; backward uniqueness; min-max game theory with both control and disturbance exercised at the interface. The latter topic also served as the PhD thesis for Jing Zhang, defended in early December 2011. Strong stability (of the original nonlinear model) at the full H^1-norm (rather than gradient norm) also served as the PhD thesis for Yongjin Lu, defended in early December 2011. Both Jing and Yongjin presently have instructorships at Virginia State University, historically an African-American college.

They are expected to receive a tenure-track assistant professorship offer in the near future.

(b) Gas flow-structure interaction. Here a structure modeling an aircraft wing sits on the (x, y)-plane, while a gas flows in the upper half-space with $z > 0$ along the x-direction at a (normalized) constant speed U. The case $0 < U < 1$ is subsonic, while the case $U > 1$ is supersonic. The gas is modeled by a linear second-order hyperbolic equation, more challenging and more complex than a classical wave-type equation, to which it reduces for $U = 0$. Interaction between the gas and the wing takes place on the wing. The wing is modeled by a nonlinear plate-type equation, possibly the von Karman equation, with clamped boundary conditions (B.C.). Problems investigated include well-posedness of the coupled gas structure interaction model at both the subsonic ($0 < U < 1$) and supersonic ($U > 1$) regimes (the latter is seriously more challenging than the former); identification of the flutter speed; elimination of flutter by introducing appropriate damping on the structure. Flutter is an intrinsic instability that may arise in certain conditions of regime. This topic also served as a PhD thesis for Justin Webster, who is expected to graduate in summer 2012.

(c) Inverse problems for PDEs. A typical illustration is as follows: A second-order hyperbolic equation is given in a multi-dimensional bounded domain Ω, with given initial conditions and a non-homogeneous boundary term (of either Neumann- or Dirichlet-type), where, however, the space-dependent coefficients or more generally the couple {damping, source}-coefficients is unknown. The goal is to 'recover' the space-dependent unknown coefficients by virtue of an additional measurement based on a boundary-trace (restriction), either of Dirichlet- or Neumann-type, respectively. The boundary measurement takes place only on a suitable, explicitly identified portion of the boundary, and over an optimal/sharp time interval, related to the finite speed of propagation of the dynamics. 'Recovery' means the two classical inverse problems of (i) 'uniqueness' and (ii) 'stability' (also called well-posedness of the nonlinear inverse problem). The above problem was studied and solved under the present project, along with the companion problems of recovery for a coupled system of two hyperbolic PDEs; the structural acoustic model; and finally, one or two coupled Schrödinger. This topic served as a PhD thesis for graduate student, Shitao Liu, defended in June 2011. Since then, Shitao holds an attractive three-year postdoctoral fellowship at the Center of

Excellence in Inverse Problems, Department of Mathematics, University of Helsinki, Finland.

(d) PDEs arising in ultrasound models in nonlinear acoustics. These PDEs are third order in time. They arise when the Fourier Law of heat-flux is substituted by the Cattaneo-Maxwell law for the purpose of avoiding the paradox of infinite speed of propagation of solutions associated to the former. Problems investigated include well-posedness by either energy or semigroup methods, structural properties, sharp stability properties, inverse problems seeking to recover one space-dependent coefficient of the equation. Presently, a graduate student of the PIs, Jason Knapp, is engaged in numerical computations with such model, as described below.

(B) Numerical Results (obtained with former PhD in AF/Academy and present PhD students).

(i) [see (D) above] **PDEs arising in ultrasound models in nonlinear acoustics.**

(i1) Numerical computations performed in collaboration with T. McDevitt (Department of Mathematical Sciences, Elizabethtown College) and R. Marchand (presently visiting the Department of Mathematics, the US Air Force Academy, Colorado) show the distribution of the spectrum (Fig. 1).

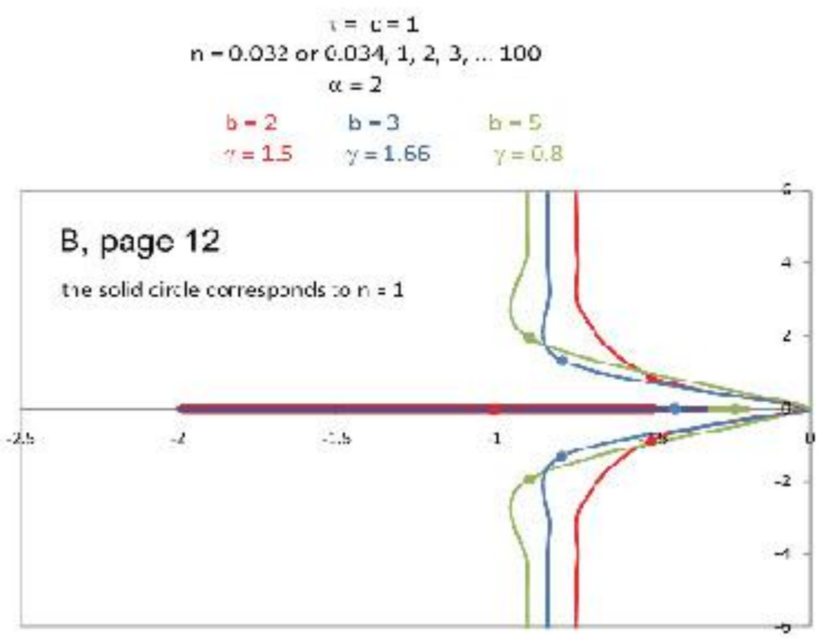

Fig. 1: Ultrasound Model: Eigenvalues; *b* Is Changing

In particular, Fig. 1 distinguishes between a component of the spectrum made of eigenvalues which are "movable" by a finite-dimensional controller, and a component (a point in the continuous spectrum), which instead is "unmovable" and depends on the properties of the fluid $-c^2/b$.

In addition, Fig. 2 shows the sensitivity of said spectrum with respect to the system parameters as it pertains to stability.

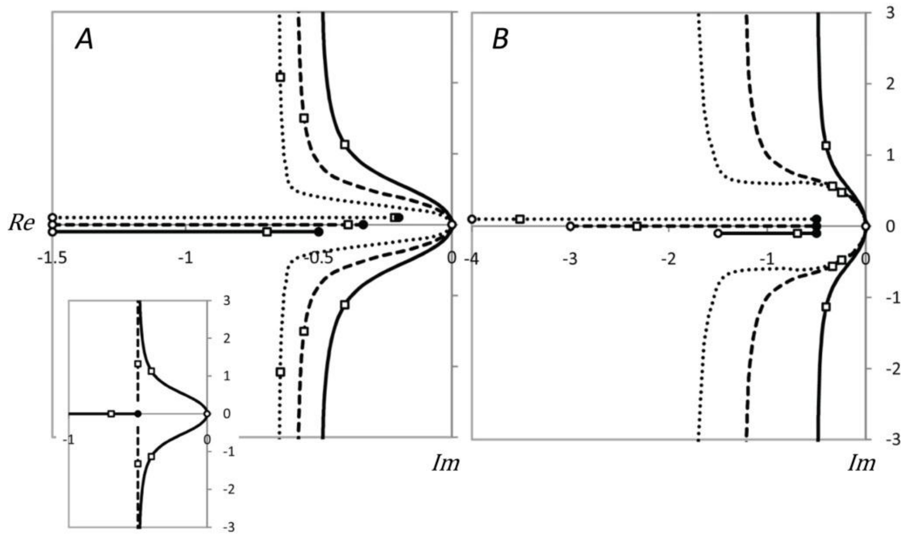

Fig. 2: Ultrasound Model: Spectrum for Different Parameters

(i2) Numerical computations performed in collaboration with Jason Knapp, a PhD student, as part of his PhD thesis. Jason is performing dynamic calculations by using the Finite Element Method. This work analyses the impact of two critical coefficients of the model on the overall stability of the system. They are the positive constant τ accounting for diffusivity (which is the coefficient of the third-derivative term), and the constant $b = \delta + \tau c^2$ (coefficient of the strong damping term), where δ is the diffusivity of the sound and *c* is the speed of sound.

(ii) Numerical computations for the model of a wave equation with acoustic boundary conditions. This is a benchmark model for all interactive structures. These computations, which also employ the Finite Element Method, were performed in collaboration with Nicolas Fourrier, and refer to issue of actuator/sensor placement for the model of the wave equation with acoustic boundary conditions. They reveal two main features: (1) The effects of over-damping, and (2) the advantage of using both boundary and interior damping, to be strategically located in order to obtain optimal design of low, as well as high, frequencies (modes), a critical goal for an infinite-dimensional system.

Fig. 3 refers to two situations under the same initial condition at the time $t = 0$ (top figures). The left column shows the evolution at $t = 1$ *interior* damping. The right column shows the evolution at $t = 1$ under *boundary* damping. The corresponding evolutions indicate that the boundary damping is almost as effective as the interior damping.

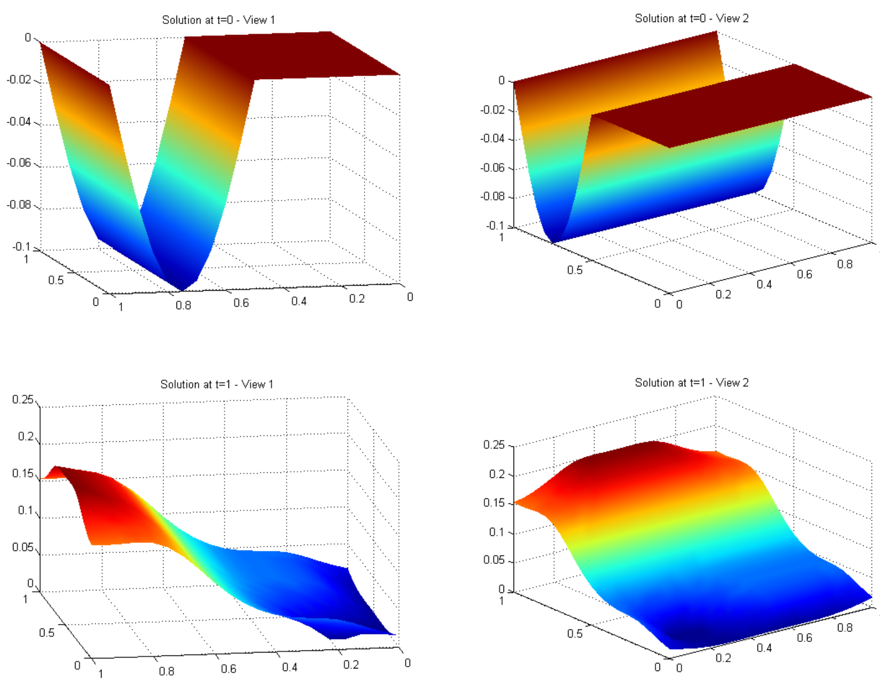

Fig. 3: Wave Equation with Acoustic Boundary Conditions.
Left Column, Interior Damping; Right Column, Boundary Damping.
Both Cases with Same Initial Condition at $t = 0$, and Corresponding Evolution at $t = 1$.

(iii) Numerical computations for models related to robotic manipulators (single link flexible robots with revolution joints). Mathematically, these are models of plates or beams with boundary dissipation (friction), however, in non-monotone boundary conditions. For this reason, they present a serious theoretical disadvantage, which makes their mathematical analysis more challenging (it involves Gevrey-type semigroups). However, non-monotone boundary conditions offer both physical and numerical advantages, as they introduce additional stability properties with a higher degree of robustness (Fig. 4a-b).

More precisely, a non-monotone, non-collocated feedback provides higher regularity properties of the dynamics than a standard monotone and collocated feedback. Thus, there is an unexpected beneficial advantage in breaking the monotonicity of the feedback, with—moreover—non-collocated feedback; that is, with control actuators placed at different boundary conditions than the damping.

The figures below offer a broad range of illustrations, such as:

- Fig. 4a considers the case *non-collocated* (non-monotone) feedback $u_{xxx} = -ku_{tx}$ on the boundary and shows more stability on high frequencies (modes) with the additional property of robustness. Refer to the `parabolic' or polynomial shape of the spectral branches, typical of Gevrey-semigroup generators. The parameter *k* is the damping parameter. It appears that $k = 1$ is the best value for stability.
- Fig. 4b considers the same case of non-collocated feedback as Fig. 4a and shows how single distinct modes change with the damping parameter *k* running from $k = 0$ to $k = \infty$; in particular, it picks the value of the parameter yielding the optimal damping for each mode.
- Fig. 5a considers the case of collocated B.C. with shear force feedback $u_{xxx} = -ku_t$ and shows the behavior of the eigenvalues for various values of the damping parameter k. High modes align toward vertical asymptote Re $\lambda = -2k$; very different behavior than the non-collocated case.

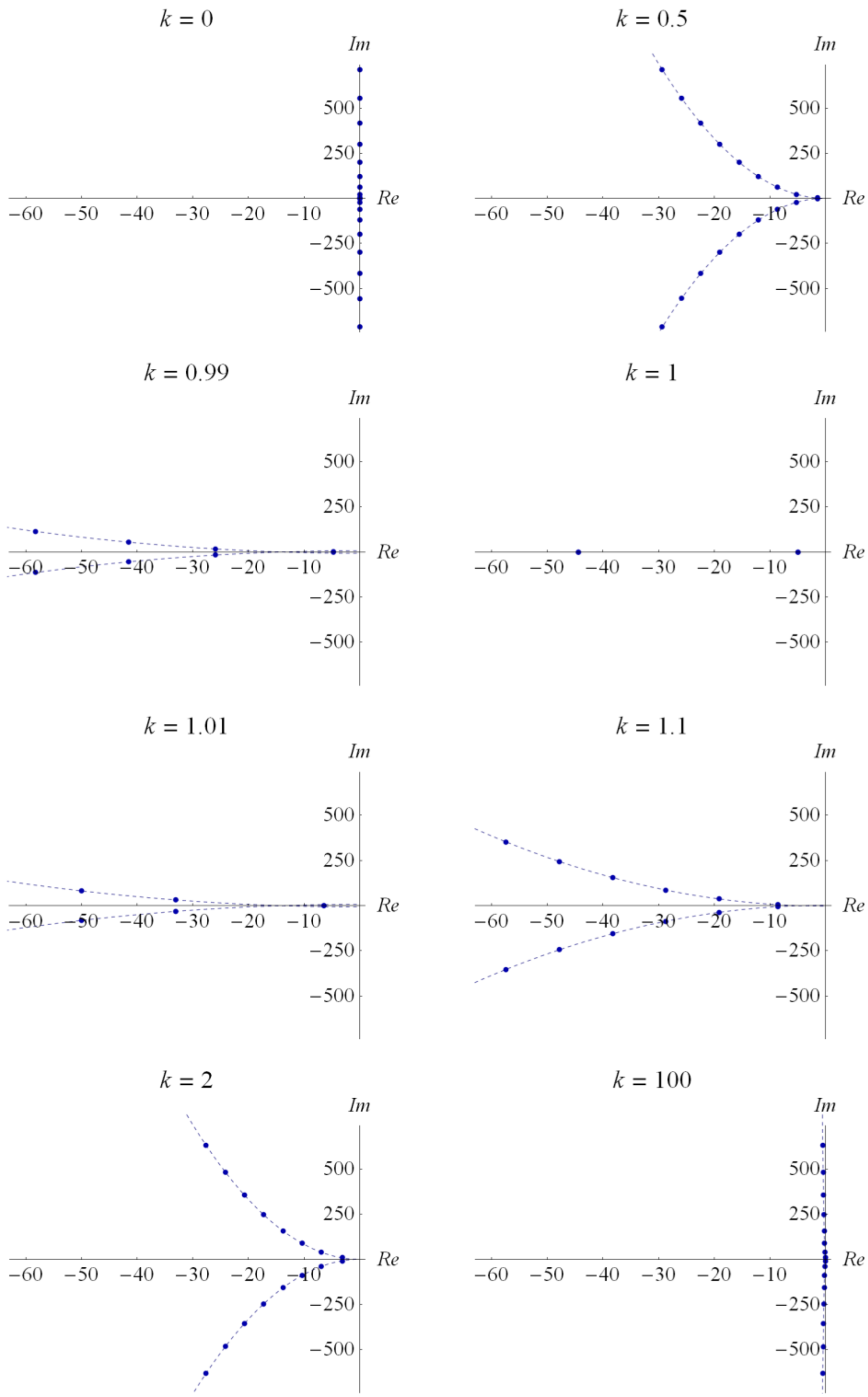

Fig. 4a: Plates/Beams with Non-Collocated, Non-Monoto0ne, Shear/Torque Boundary Feedback $u_{xxx} = -ku_{tx}$. Spectrum of Generator for Different Values of *k*.

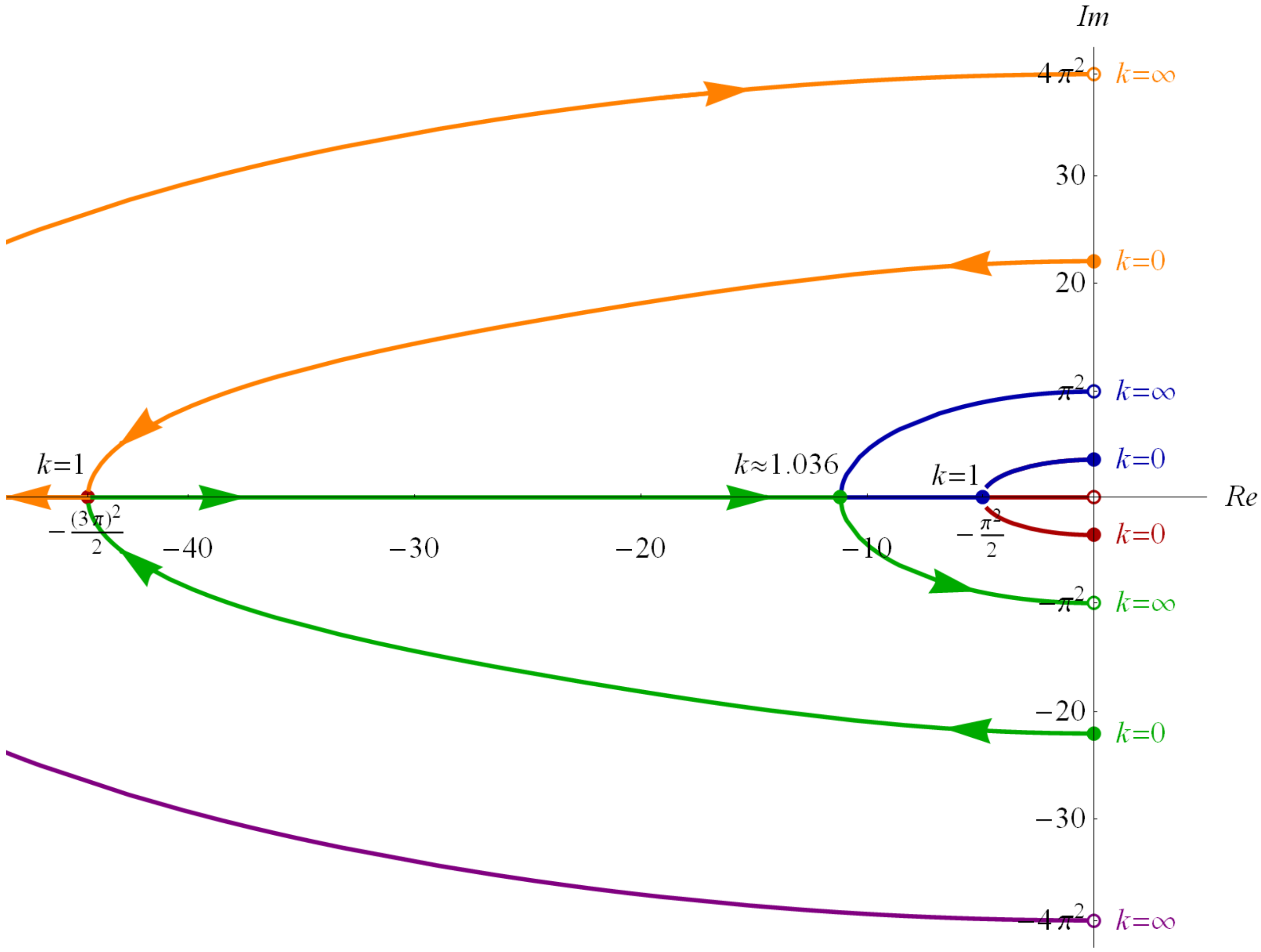

Fig. 4b: Plates/Beams with Non-Collocated, Non-Monoto0ne, Shear/Torque Boundary Feedback $u_{xxx} = -ku_{tx}$. Variation of Single Modes with *k*. Best Value of *k* for Stability of Each Single Mode.

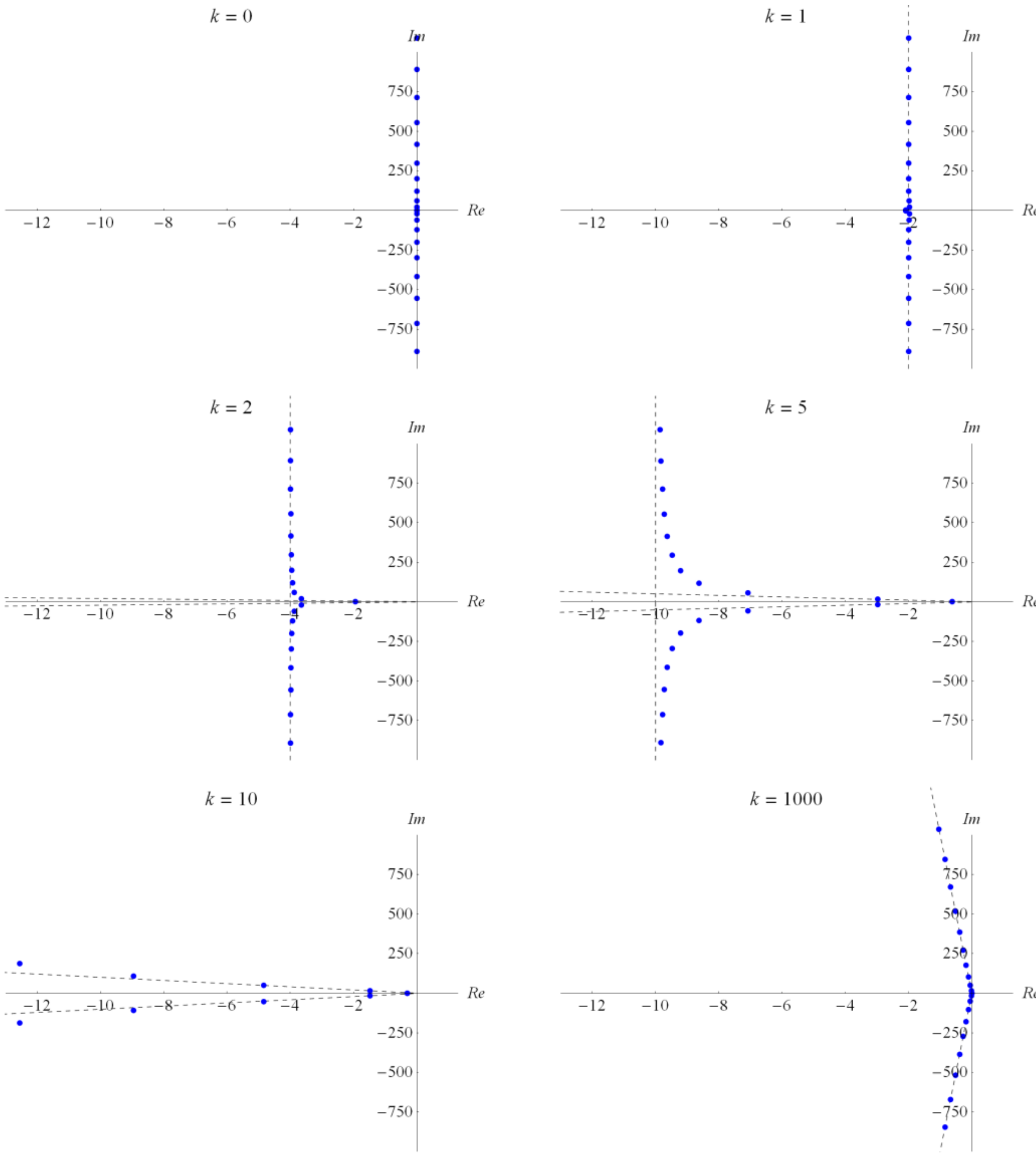

Fig. 5a: Plates/Beams with Collocated Shear/Torque Boundary Feedback $u_{xxx} = -ku_t$.
High Modes Align toward Vertical Asymptote Re $\lambda = -2k$,
Unlike Non-Collocated Case.

- Fig. 5b considers the case of collocated shear/force boundary feedback as Fig. 5a and shows how single distinct modes change with the damping parameter *k* running from $k = 0$ to $k = \infty$; in particular, it picks the value of the parameter *k* yielding the optimal damping for each mode.

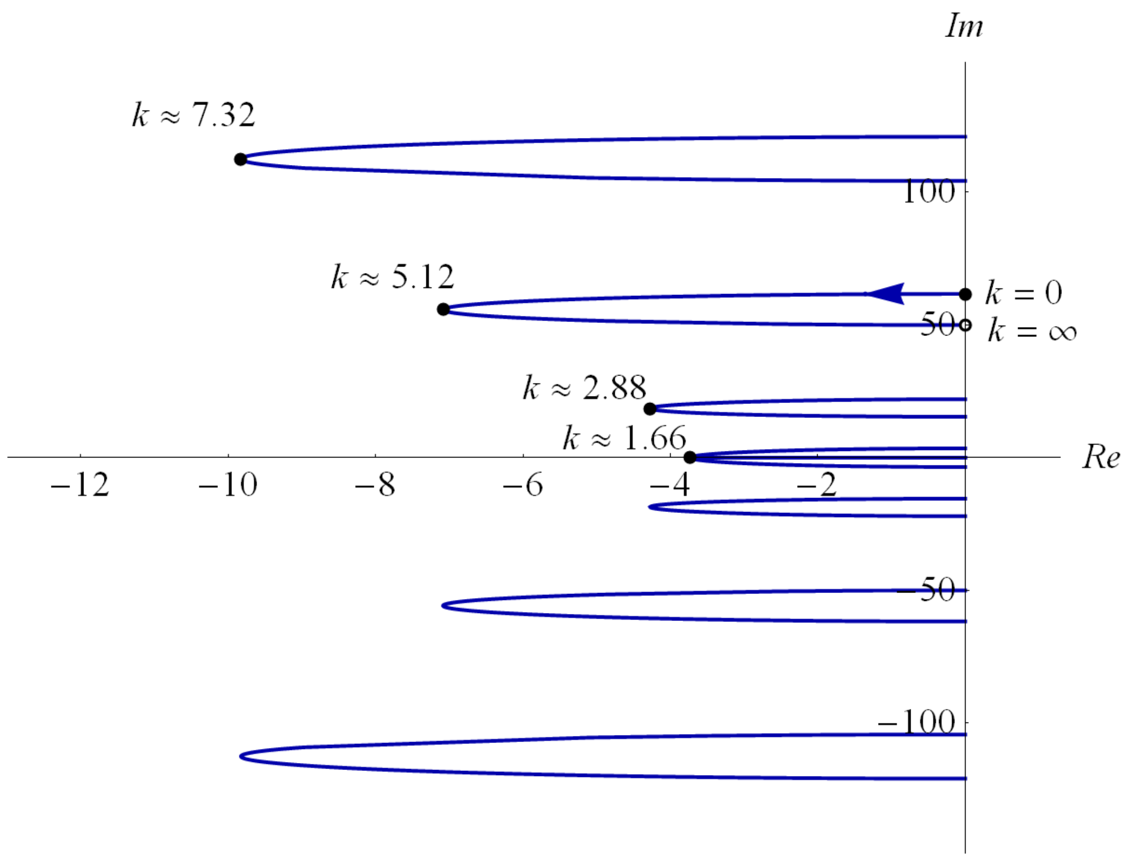

Fig. 5b: Plates/Beams with Collocated Shear/Force Boundary Feedback $u_{xxx} = -ku_t$. Variation of Single Modes with *k*. Best Value of *k* for Stability for Each Single Mode.

- Figure 6a considers the case of collocated moment/torque boundary feedback $u_{xx} = -ku_{tx}$ and shows the behavior of the eigenvalues for various values of the damping parameter *k.* High modes align toward vertical asymptote Re $\lambda = -1/k$. Thus, *k* large moves asymptote toward Im-axis.

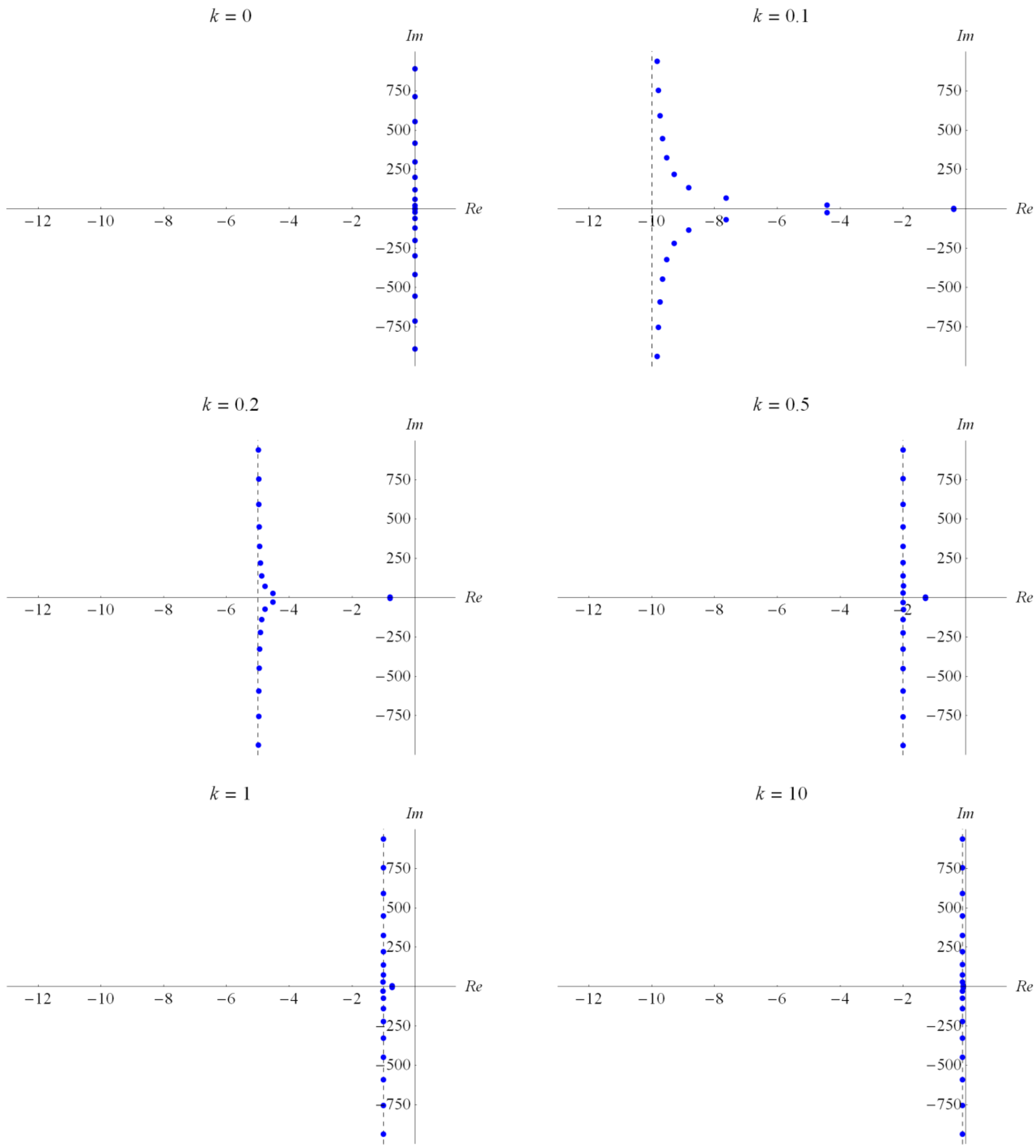

Fig. 6a: Plates/Beams with Collocated Moment/Torque Boundary Feedback $u_{xx} = -ku_{xt}$. Variational Single Modes with k. High Modes Align toward Vertical Asymptote Re $\lambda = -1/k$. Large k, Asymptote Closer to Im-Axis.

- Fig. 6b considers the same case of collocated moment/torque boundary feedback as Fig. 6a and shows how single eigenvalues change with the damping parameter k running from $k = 0$ to $k = \infty$; in particular, it picks the value of the parameter k yielding the optimal damping for each mode. For $k = 0, \infty$: no damping.

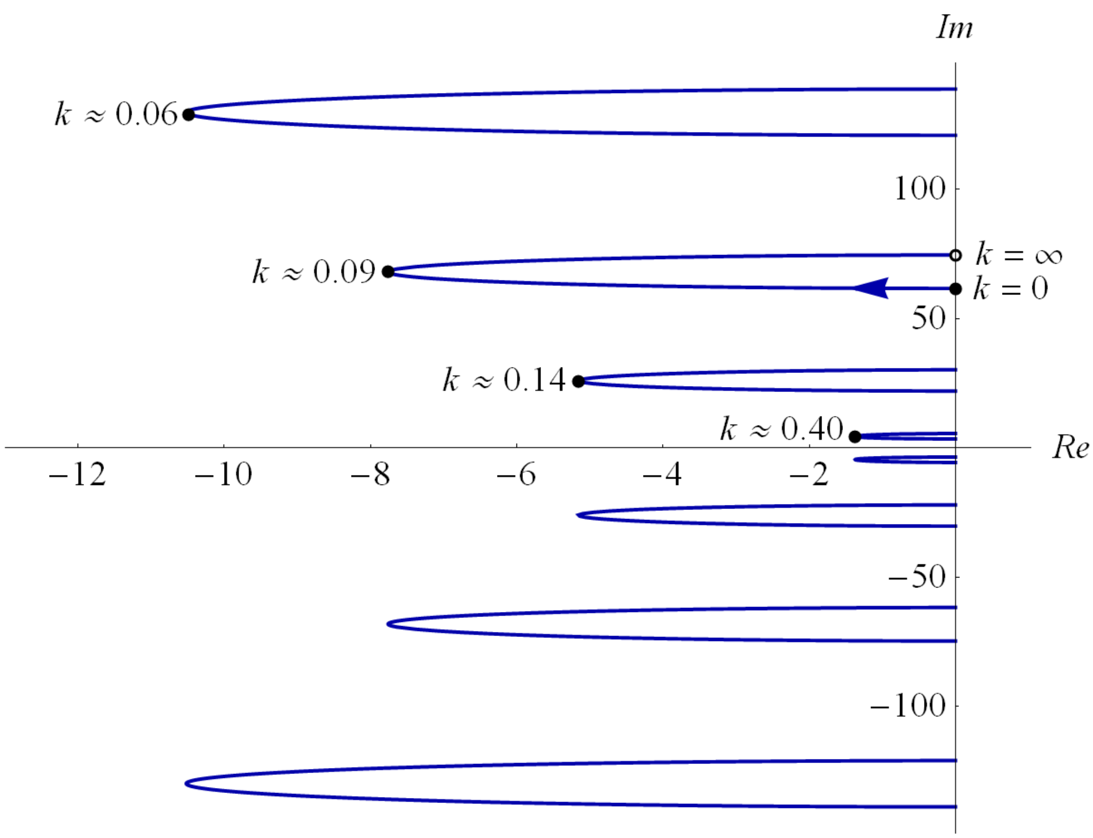

Fig. 6b: Plates/Beams with Collocated Moment/Torque Boundary Feedback $u_{xx} = -ku_{xt}$. Variation of Single Modes with k. For $k = 0, \infty$, No Stability. Best Value of k for Stability of Each Single Mode.